$$\int \frac{x}{\sqrt{2+x}}dx = \int x(2+x)^{-\frac{1}{2}}dx = x\cdot 2(2+x)^{\frac{1}{2}} - \int 2(2+x)^{\frac{1}{2}}dx = 2x(2+x)^{\frac{1}{2}} - 2\int (2+x)^{\frac{1}{2}}dx =$$

$$u = x \Rightarrow du = dx$$

$$dv = (2+x)^{-\frac{1}{2}}dx \Rightarrow v = \int (2+x)^{-\frac{1}{2}}dx = \int z^{-\frac{1}{2}}dz = \frac{z^{\frac{1}{2}}}{\frac{1}{2}} = 2z^{\frac{1}{2}} = 2(2+x)^{\frac{1}{2}}$$

7.

$$z = (2+x) \Rightarrow dz = dx$$

$$2x(2+x)^{\frac{1}{2}} - 2\int u^{\frac{1}{2}}du = 2x(2+x)^{\frac{1}{2}} - 2\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = 2x(2+x)^{\frac{1}{2}} - \frac{4}{3}u^{\frac{3}{2}} = 2x(2+x)^{\frac{1}{2}} - \frac{4}{3}(2+x)^{\frac{3}{2}} =$$

$$u = (2+x) \Rightarrow du = dx$$

$$2(2+x)^{\frac{1}{2}}\left(x - \frac{2}{3}(2+x)\right)$$

8.

$$\int \frac{x^2}{\sqrt{2+x}}dx = \int x^2(2+x)^{-\frac{1}{2}}dx = x^2\cdot 2(2+x)^{\frac{1}{2}} - \int 2(2+x)^{\frac{1}{2}}\cdot 2xdx = 2x^2(2+x)^{\frac{1}{2}} - 4\int x(2+x)^{\frac{1}{2}}dx$$

$$u = x^2 \Rightarrow du = 2xdx$$

$$dv = (2+x)^{-\frac{1}{2}}dx \Rightarrow v = \int (2+x)^{-\frac{1}{2}}dx = \int z^{-\frac{1}{2}}dz = \frac{z^{\frac{1}{2}}}{\frac{1}{2}} = 2z^{\frac{1}{2}} = 2(2+x)^{\frac{1}{2}}$$

$$z = (2+x) \Rightarrow dz = dx$$

$$2x^2(2+x)^{\frac{1}{2}} - 4\left[x\cdot\frac{2}{3}(2+x)^{\frac{3}{2}} - \int \frac{2}{3}(2+x)^{\frac{3}{2}}dx\right] = 2x^2(2+x)^{\frac{1}{2}} - 4\left[x\cdot\frac{2}{3}(2+x)^{\frac{3}{2}} - \frac{2}{3}\int (2+x)^{\frac{3}{2}}dx\right] =$$

$$u = x \Rightarrow du = dx$$

$$dv = (2+x)^{\frac{1}{2}}dx \Rightarrow v = \int (2+x)^{\frac{1}{2}}dx = \int z^{\frac{1}{2}}dz = \frac{z^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}z^{\frac{3}{2}} = \frac{2}{3}(2+x)^{\frac{3}{2}}$$

$$z = (2+x) \Rightarrow dz = dx$$

$$2x^2(2+x)^{\frac{1}{2}} - 4\left[x\cdot\frac{2}{3}(2+x)^{\frac{3}{2}} - \frac{2}{3}\int u^{\frac{3}{2}}dx\right] = 2x^2(2+x)^{\frac{1}{2}} - 4\left[x\cdot\frac{2}{3}(2+x)^{\frac{3}{2}} - \frac{2}{3}\cdot\frac{u^{\frac{5}{2}}}{\frac{5}{2}}\right] =$$

$$u = (x+5) \Rightarrow du = dx$$

$$2x^2(2+x)^{\frac{1}{2}} - 4\left[x\cdot\frac{2}{3}(2+x)^{\frac{3}{2}} - \frac{4}{15}\cdot(2+x)^{\frac{5}{2}}\right] = 2x^2(2+x)^{\frac{1}{2}} - \frac{8}{3}x(2+x)^{\frac{3}{2}} + \frac{16}{15}(2+x)^{\frac{5}{2}} =$$

$$2(2+x)^{\frac{1}{2}}\left(x^2 - \frac{4}{3}x(2+x) + \frac{8}{15}(2+x)^2\right)$$

9.

$$\int \frac{x}{\sqrt{2+x^2}}dx = \int x(2+x^2)^{-\frac{1}{2}}dx = \frac{1}{2}\int u^{-\frac{1}{2}}du = \frac{1}{2}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} = u^{\frac{1}{2}} = \left(2+x^2\right)^{\frac{1}{2}}$$

$$u = \left(2+x^2\right) \Rightarrow du = 2xdx \Rightarrow \frac{1}{2}du = xdx$$

10.

$$\int (x-1)^2 \ln x dx = \ln x\left(\frac{x^3}{3} - x^2 + x\right) - \int\left(\frac{x^3}{3} - x^2 + x\right)\frac{1}{x}dx = \ln x\left(\frac{x^3}{3} - x^2 + x\right) - \int\left(\frac{x^2}{3} - x + 1\right)dx =$$

$$u = \ln x \Rightarrow du = \frac{1}{x}dx$$

$$dv = (x-1)^2 dx \Rightarrow v = \int (x-1)^2 dx = \int (x^2 - 2x + 1)dx = \frac{x^3}{3} - \frac{2x^2}{2} + x = \frac{x^3}{3} - x^2 + x$$

$$\ln x\left(\frac{x^3}{3} - x^2 + x\right) - \left[\frac{1}{3}\int x^2 dx - \int xdx + \int dx\right] = \ln x\left(\frac{x^3}{3} - x^2 + x\right) - \frac{1}{3}\cdot\frac{x^3}{3} - \frac{x^2}{2} + x =$$

$$\ln x\left(\frac{x^3}{3} - x^2 + x\right) - \frac{1}{9}x^3 - \frac{x^2}{2} + x$$

11.

$$\int e^x \cos xdx = e^x \cdot \sin x - \int \sin x \cdot e^x dx = e^x \sin x - \int e^x \sin xdx =$$

$$u = e^x \Rightarrow du = e^x dx$$

$$dv = \cos xdx \Rightarrow v = \int \cos xdx = \sin x$$

$$e^x \sin x - \left[e^x \cdot -\cos x - \int e^x \cdot -\cos xdx\right] = e^x \sin x + e^x \cos x - \int e^x \cos xdx$$

$$u = e^x \Rightarrow du = e^x dx$$

$$dv = \sin xdx \Rightarrow v = \int \sin xdx = -\cos x$$

Since we are back to the original integral equate original question with possible answer:

$$\int e^x \cos xdx = e^x \sin x + e^x \cos x - \int e^x \cos xdx$$

$$2\int e^x \cos xdx = e^x \sin x + e^x \cos x \Rightarrow \int e^x \cos xdx = \frac{e^x}{2}\left(\sin x + \cos x\right)$$

12.

$$\int \cos(\ln x)dx = \cos(\ln x)\cdot x - \int x \cdot -\sin(\ln x)\cdot \frac{1}{x}dx = \cos(\ln x)\cdot x + \int \sin(\ln x)dx =$$

$$u = \cos(\ln x) \Rightarrow du = -\sin(\ln x)\cdot \frac{1}{x}dx$$

$$dv = dx \Rightarrow v = \int dx \Rightarrow v = x$$

$$\cos(\ln x)\cdot x + \left[\sin(\ln x)\cdot x - \int x\cdot\cos(\ln x)\cdot\frac{1}{x}dx\right] = \cos(\ln x)\cdot x + \sin(\ln x)\cdot x - \int \cos(\ln x)dx$$

$$u = \sin(\ln x) \Rightarrow du = \cos(\ln x)\cdot \frac{1}{x}dx$$

$$dv = dx \Rightarrow v = \int dx \Rightarrow v = x$$

The question forms a loop - original question part of calculated answer $\therefore$

$$\int \cos(\ln x)dx = \cos(\ln x)\cdot x + \sin(\ln x)\cdot x - \int \cos(\ln x)dx$$

$$2\int \cos(\ln x)dx = \cos(\ln x)\cdot x + \sin(\ln x)\cdot x \Rightarrow \int \cos(\ln x)dx = \frac{1}{2}\left(\cos(\ln x)\cdot x + \sin(\ln x)\cdot x\right)$$